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Received: 03-May-2023, Manuscript No: jaet-23-96291; **Editor assigned:** 05-May-2023, Pre-QC No: jaet-23-96291 (PQ); **Reviewed:** 19-May-2023, QC No: jaet-23-96291; **Revised:** 22-May-2023, Manuscript No: jaet-23-96291 (R); **Published:**

viewpoint. e present work represents the rst preliminary attempt

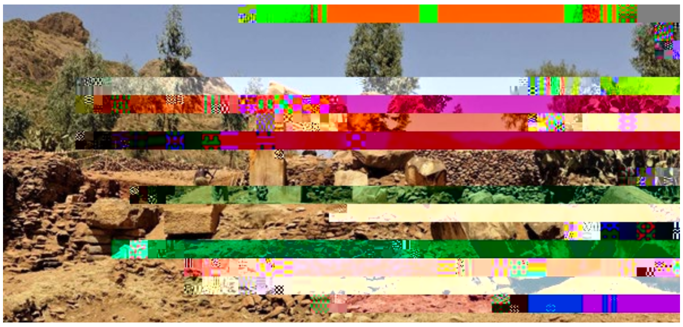

Figure 2: Photo from the site of archaeological excavation of the Grat Be'al Gibri.

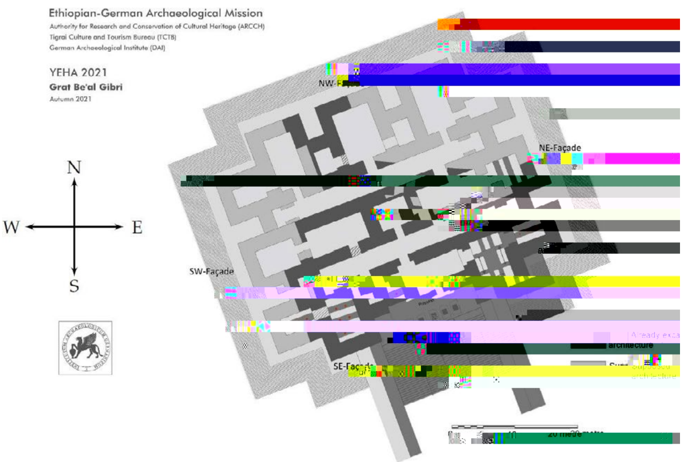

Figure 3: Reconstructed ground plan after excavations **[8]**.

the interior of this type of building is divided into numerous rooms per story (Figure 3), with several stories being assumed in each case. The lower part of this type of building usually consists of a podium, which could be considered as a foundation, above which the actual floor building rises.

The 6 m high podium at the Grat Be'al Gibri in Yeha consists of a chamber wall system, whereby the chambers are filled with quarry stone material. The ground plan of the building, both the podium and the superstructure, is approximately square, with the four sides each divided by three tower-like projecting risalites. The fact that the corner risalites are assigned to the respective sides creates indented corners. The central risalit of the SE facade is designed as a monumental propylon with six monolithic pillars and thus marks the only verifiable entrance to date. Passing the propylon the entrant reaches a monumental doorway with door jambs. In front of the propylon is a wide flight of steps, which is bordered on both sides by two flanking walls.

The enormous thickness of the podium walls of 2.20 m and the walls above of 1.90 m together with the finding of a stairtower were the starting point for considerations that it must have been a multi-story building. This assumption is supported by the height of the pillars of the propylon marking the main entrance, which can be reconstructed as around 10 m on the basis of comparisons of proportions. Since the pillars were covered with architraves, on which in turn ceiling beams lay that connected the propylon with the main building, the latter must have been at least this height, i.e. about three stories high. Ancient rock and wall paintings depict multi-story buildings with a much higher number of stories [**9, 10**].

Since there are no further findings on the structure regarding the number of stories to be reconstructed, the building is shown in a first draft with five standard stories and above them three additional, recessed stories (Figure 4). These recessed upper three stories are also visible on rock paintings and murals in South Arabia [**9, 10**]. Since the pillar propylon in the area of the front façade corresponds exactly to the shape of a central risalit both in its depth and width, the area above the propylon is modelled as a central risalit over two additional stories, as on the other sides of the façade. In the south-east of the building, remains of a surrounding stepped embankment made of stones, called glacis, have been preserved, which surrounded in the reconstruction the entire building for reasons of symmetry, except in the area of the monumental flight of steps.

2.1.3 Construction and material

The archaeological-architectural investigations at the palace building Grat Be'al Gibri in Yeha, in cooperation with geologists and archaeobotanists, led to the identification of various construction materials used [**12**]. In the case of the walls, those below the ground floor can be distinguished from those above: For the walls of the podium serving as a foundation under the ground floor, quarry stones of locally available phonolite were used, which were masoned with a solid yellow clay mortar, also locally available, to form a quarry stone masonry. Neither continuously horizontal courses nor masonry shell structures can be observed, which is probably due to the polygonal fracture behavior of the phonolite stone. For the construction of the timber-reinforced walls above the podium, hardwoods (African Olive and Cordia Africana) were used in addition to the clay-mortared rubble stone masonry. The timbers, all axed into rectangular beams, were horizontally placed in the wall in longitudinal and transversal direction as shown in Figure 5. Cross-sections of the beams of 0.21 m - 0.28 m can be reconstructed on the basis of beam imprints and cavities in the masonry, with the majority of the imprints having edge lengths of around 0.24 m. In the 3D CAD reconstruction, the latter dimension has been used as the edge length of all beams. The lowest timber layers always run transversely to the wall axis, followed by beams in the longitudinal direction. The two outer beams in the longitudinal axis are flush with the outer edge of the wall.

Juniper (*Juniperus procera*) was used as ceiling-supporting columns within the rooms. The round or faceted columns, up to 0.40 m in diameter, stood in the rooms with spans of less than 2 m, which indicates enormous ceiling loads. The construction of the ceilings is not certain.

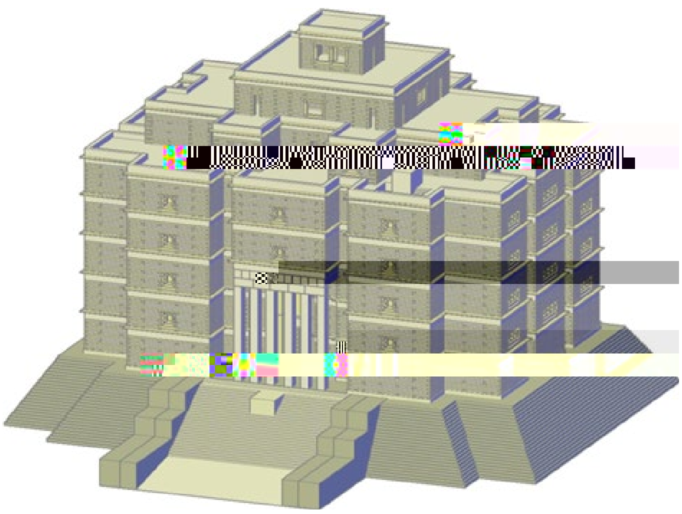

Figure 4: 3D geometrical (CAD) model according to the reconstruction in **[11]**.

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It would be conceivable to have co ered or lath ceilings, of which there are at least late antique examples in South Arabia and East Africa [**13, 14**]. Since only crumbled and heavily charred remains of beams have preserved in the debris of the rooms, it is di cult to determine the type of wood used for the ceilings. However, it seems that the ceilings consisted mainly of hardwoods (see above) and occasionally Juniper.

ere is no archaeological evidence for the windows shown in the 3D reconstruction. It cannot be ruled out that - as shown there - the ground oor was without windows at all. At least for the upper oors windows can be assumed. In the absence of evidence, they are reconstructed as an imitation of known Aksumite windows, but again avoiding vertically arranged wooden elements.

All elements of the propylon and the subsequent door system are made of local sandstone elements. is was processed by stonemasons at great expense - until a nal grinding was made. e propylon pillars were made of monoliths, probably also the door jambs. Both rest on massive bases, which were also made of sandstone. e large sandstone fragments found in the debris in front of the main entrance belong on the one hand to the architraves connecting the propylon pillars together and on the other hand to the ceiling beams above connecting the propylon with the main building.

2.2 3D ni e elemen model

In this Section, the geometrical and architectural model of the building is transformed into a numerical simulation model within the nite element program ANSYS. At that, some simpli cations have been taken as described below. e description is divided into geometrical parameters in Section 2.2.1, material parameters in Section 2.2.2, boundary conditions in Section 2.2.3 and model parameters in Section 2.2.4. e e ects of the present simpli cations on the obtained results are discussed later in Section 4.

2.2.1 Geome ical pa ame e

e oor plan of the palace Grat Be'al Gibri is assumed to be axisymmetric at each story. In Figure 6, the oor plan at story 4 is shown with a de ned global cartesian coordinate system. Instead of de ning a global regular grid, local axes have been de ned for each individual wall according to its longitudinal and transversal directions. e origin of the de ned system is located in the intersection of the symmetry axes. e horizontal axes are indicated by numbers, positive above and negative below the global -axis. Due to the symmetry, the axes with the same absolute number have the same distance from the global -axis. e vertical axes are marked with letters. e uppercase letters below the oor plan refer to the major vertical axes, the lowercase letters above the oor plan to the minor vertical axes of walls, which are only between the horizontal axes −2 and 2. e appended abbreviation in Figure 6 indicates whether the axis is le (L/l) or right (R/r) of the global -axis.

e outer dimensions are × = 47.52 m × 37.44 m in story 1 to 5. As mentioned in Section 2.1.2, the thickness of the walls varies between $_{HW,0}$ = 2.16 m in story 0 to $_{HW,8}$

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- No consideration of stairs including the stair tower
- No consideration of supporting columns inside the building
- No consideration or enlargement of wall openings (e.g. for doors)
- Small adapting/shi ing of wall axes
- No consideration of decorative elements (e.g. corbels, attic)
- No consideration of the chamber lling in the podium as well as the outer ll/staircase structure (Glacis)

us, a numerical model with a mostly regular nite element mesh, highest possible accuracy and acceptable costs for the computation could be created.

2.2.2 Ma e ial pa ame e

Phonoli e and imbe -cla -compo i e fo all

e rising walls of the building consist of clay-mortared, timber-reinforced quarry stone masonry. us, it is a composite wall whose load-bearing behavior depends on the properties of its constituents and their interaction. Such composites can be modeled in detail with enormous computation cost if local interaction mechanisms are of interest. Alternatively, they can be homogenized if the focus is set onto the overall behavior. In this study, the wall structure has been homogenized for simplicity. e underlying material parameters of phonolite and the timber-clay-composite are described in the following.

For the material parameters of phonolite, it is only referred to phonolite rocks [**12**]. is allows the assumption that it is not a tu , but for the sake of completeness this is not excluded. In the literature, no source could be found in which mechanical parameters for phonolite are listed, whereas values for plutonite, vulcanites and tu s could be found in di erent sources, see Table 1. Due to the classi cation of phonolite as vulcanite, these values can be regarded as appropriate limits until more precise ndings are available.

Based on Table 1, the following minimum, mean and maximum values for the material parameters of phonolite have been used in the numerical simulations:

$$\begin{aligned} &\text{Density } {}_{Ph} = \{1.30 \quad 2.25 \quad 3.20\}\ t/m^3,\\ &\text{Young's modulus } {}_{Ph} = \{20 \quad 65 \quad 110\}\ GPa,\\ &\text{Compressive strength } {}_{c,Ph} = \{5 \quad 202.5 \quad 400\}\ MPa,\\ &\text{Tensile strength } {}_{t,Ph} = \{4 \quad 17 \quad 30\}\ MPa \text{ and}\\ &\text{Flexural strength } {}_{f,Ph} = \{1 \quad 35.5 \quad 70\}\ MPa. \end{aligned} \tag{1}$$

e timber-clay-composite is simpli ed to one material with the following mean values based on [**19**]:

$$\begin{aligned} &\text{Density } {}_{TC} = 1.638\ t/m^3 \text{ and}\\ &\text{Compressive strength } {}_{c,TC} = 4.62\ MPa. \end{aligned} \tag{2}$$

Consequently, the material properties of the homogenized wall structure are de ned as:

$$\begin{aligned} &\text{Density } {}_{HW} = (1 - {}_{TC}) \ {}_{Ph} + {}_{TC} \ {}_{TC} \text{ with}\\ &\text{Timber-clay percentage } {}_{TC} = \{0 \quad 1\},\\ &\text{Young's modulus } {}_{HW} = {}_{Ph} \text{ and}\\ &\text{Poisson's ratio } {}_{HW} = 0.25\ [20]. \end{aligned} \tag{3}$$

nipe imbe fo ceiling

A density of ${}_{JT}$ = 0.6 t / m^3 and values for the compressive and exural strength for juniper timber can be found in [**21**]. However, juniper is a cypress species among the conifers. Since the ceiling is relevant in this study mainly due to its dead load, the mentioned value for the density is used, but strength values of an equivalent C24

Rock	Density	Young's modulus	Compressive strength	Tensile strength	Flexural strength
	[t / m³]	[GPa]			

so wood parallel to the bre direction are applied (cf. [**22**]), leading to:

Density $_{JT}$ = 0.6 t ⁄ m^3,

Young's modulus $_{JT}$ = 11 GPa,

Poisson's ratio $_{JT}$ = 0 (for simplicity), (4)

Compressive strength $_{c,JT}$ = 21 MPa,

Tensile strength $_{t,JT}$ = 14 MPa and

Flexural strength $_{f,JT}$ = 24 MPa.

Sand . one fo pilla p op lon

Local sandstone was used for the pillar propylon. Its material properties have been analyzed in a similar way as for the phonolite and have been summarized in Table 2.

Based on Table 2, the following minimum, mean and maximum values for the material parameters of sandstone have been used in the numerical simulations:

density $_{Ss}$ = {2.00 2.35 2.70} t ⁄ m^3,

Young's modulus $_{Ss}$ = {5 37.5 70} GPa,

Poisson's ratio $_{Ss}$ = 0.25 [20], (5)

compressive strength $_{c,Ss}$ = {15 152.5 290} MPa,

tensile strength $_{t,Ss}$ = {20 22.5 25} MPa and

exural strength $_{f,Ss}$ = {3 18 33} MPa

2.2.3 B nda condi. ion

Di ichle. b nda condi. ion

e focus of the present study is put on the superstructure of

3. Re l.

e results of representative numerical simulations of the building are shown and analyzed in this Section. ey result from the geometrically and physically linear static analysis. e focus is on the displacement eld) and on stresses at critical locations within the structure. e maximum rst and the minimum third principal stress ($_{1,max}$ $_{3,min}$) are compared regarding various input parameters. Especially for some material parameters, an extremely large uncertainty has been detected in Section 2.2.2. For simplicity, the timber-clay percentage is set to $_{TC}$ = 0 in all simulations. A study concerning the heterogeneous wall structure is essential, but beyond the scope of this paper.

e results using mean values for density and Young's modulus for phonolite as well as for sandstone are given in Section 3.1. In Section 3.2, the variation of the densities is conducted and analyzed. Furthermore, the Young's modulus is varied in Section 3.3.

3.1 S al a e men. b e of mean e fo ma. e ial pa ame. e

e used mean values for the numerical simulation are listed in Table 3. e resulting displacement eld is displayed in Figure 8 and Figure 9.

e maximum absolute horizontal displacement occurs at ground level with a value of $_{H,max}$ = 0.0468 mm. e displacement values and under dead load are, as excepted, very small and nearly double symmetrical. Also the vertical displacements are small and nearly double symmetrical with a maximum absolute value on the top of $_{V,max}$ $_{,max}$ = 0.175 mm. It is worth mentioning that the homogenized material properties generally lead to a higher sti ness of the structure and smaller displacements compared to that with inhomogeneous material. Another reason for small displacements is neglecting soil and structural settlements, so that the obtained displacements represent only compressive deformations within the structure.

e stress state of the structure can be well characterized by the principal stresses in each element. e third principal stress $_{3}$, which corresponds mainly to the vertical compressive stress , is depicted in Figure 10.

e minimum value of $_{3,\ min,Sing}$ = −1.08 MPa is located between story 1 and 0 caused by a geometrical singularity. Excluding this region, the minimum value of $_{3,min,Unsc}$ = −0.66 MPa is, as assumed, at ground level in the wall, see Figure 10 (Right). e stress value has to be scaled

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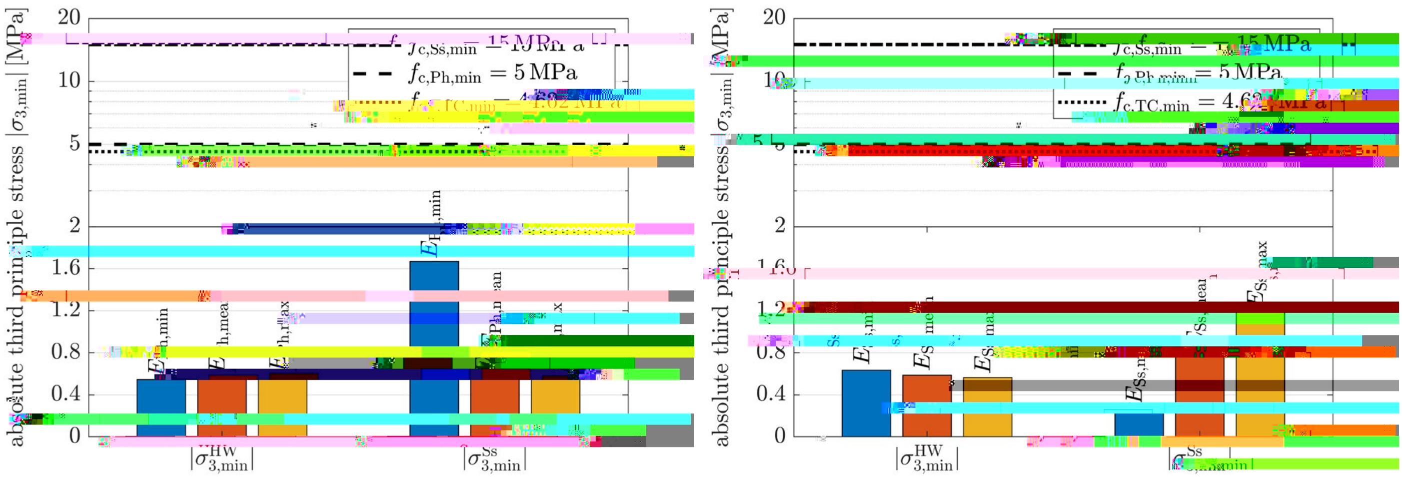

Figure 13: Comparison of the minimal third principal stresses for the variation of the Young's modulus. Left: $_{Ph}$ of the wall (100 % phonolite). Right: $_{Ss}$ of the pillar propylon (sandstone).

construction, since the ceiling performance should become a matter of the next detailed study.

e further simpli cation has been made that the building is loaded mainly by the self-weight, since this parameter can be estimated quite well from the geometrical dimensions of structural members and their density. e wind load is almost irrelevant for such a massive structure. e live loads from persons, equipment and storage are static in nature and can be taken in the rst step as a portion of the dead load like in the present study. Dynamic loads, for example by the earthquakes, are out of the scope of the present study.

Finally, the staircase tower has been excluded from the model which does not in uence the structural behavior since it is independent of the remaining building.

e structural performance has been decided to assess by means of principal stresses in critical components and structural deformations as a whole. It is consistent with the assumptions made above and can lead to the rst reasonable estimations. e principal stresses, even if they are determined within the linear elastic analysis, can show how far the stress state is from the critical one or from the limits of the load-bearing capacity.

e obtained results allow the following general conclusions. e principal stresses are mainly compressive ones resulting from the self-weight of the structure. e maximum magnitude of $_3$ = 1.08 MPa is far below the minimum compressive strength for masonry $_{c,TC,min}$ = 4.62 MPa. Even if the live load of the building is set equal to the dead load, i.e. the dead load is doubled, the compressive stress lies still far below the relevant strength. Whether the same compressive stress (bearing pressure) is critical for the soil underneath the building, cannot be answered without a soil inspection and characterization on site.

Since the tensile strength in masonry is generally quite low, it is usually not considered as a relevant design characteristics in civil engineering. High tensile stresses can lead to tensile or shear cracks in masonry. Some constructional measures should be taken to carry tensile stresses, for example, reinforcing. e maximum tensile stress $_1$ = 0.51 MPa in the palace construction is determined to be directed in the horizontal - plane. e horizontal placement of timber beams seems to ful l just this reinforcing function with respect to horizontal tensile stresses. For comparison purposes only, the tensile strength of ordinary low-strength concrete is about $_{ctm}$ = 1.60 MPa and that of clay is about 0.05 MPa. e tensile stresses in the present case are between these values, so the crack appearance seems to be probable and the reinforcement reasonable.

In this context, it is worth mentioning that the maximum compressive and tensile stresses occur in the middle part of the building. e outer perimeter is composed of three closed cells along each side of the ground plan (Figure 3), which lead to the tower-like construction for each cell over the height. It is likely that the ground plan with outer cells was selected similar to the honeycomb structures. e advantage of that choice is now numerically approved for the stresses. e stresses in these cells are essentially smaller compared to those in the inner walls in Figure 10 (Right). Besides, the stability of the connected tower group would be essentially higher than that of a building with smooth and long walls, albeit quite thick walls.

e present virtual building possesses ve regular stories. Summarizing the obtained numerical results and taking into account involved uncertainties, it can still be concluded that the original building could certainly have several stories. Whether their amount is smaller or larger than ve, needs additional investigation, including the soil and the composite structure in detail.

Some of the archaeological questions about the building cannot be answered yet; they should be a matter of further investigation. Among them, the role of the timber beams in the composite wall structure including their interaction with stones and clay, the structural performance of ceilings under local bending and their bearing capacity, the soil-structure interaction and its in uence on the global deformation and load-bearing capacity, the in uence of structural details like openings and so on. e answers to these questions require multi-scale modeling techniques and more information about speci c features of used materials and members. Since this information is very limited or even unavailable, advanced non-deterministic simulation methods should be applied to properly quantify uncertainty and reduce it by use of special laboratory experiments on representative reference constructions as well as measurements on site. The uncertainty quantification was the topic of the DFG Priority Program "Polymorphic uncertainty modelling for the numerical design of structures – SPP 1886". The authors are going to apply the corresponding experience with probabilistic and non-probabilistic simulation methods (cf. [**23,24**]) onto the virtual reconstruction of the palace building in Yeha and perform the above mentioned investigation steps in the near future.

Acknowledgements

The archaeological and building historical investigations within the Ethiopian-German cooperation-project in Yeha are funded since 2016 by the DFG (*Deutsche Forschungsgemeinschaft*); grant number: GE 1980/4-1.

References

1. Gerlach, I. Cultural Contacts between South Arabia and Tigray (Ethiopia) during the early 1st millennium BC. Results of the Ethiopian-German Cooperation Project in Yeha. Zeitschrift für Orient-Archäologie 2013, 6, 252–275.

2. Schnelle, M. Observations on architectural features from the early 1st millennium BC in South Arabia and East Africa. In Contacts between South Arabia and the Horn of Africa, from the Bronze Age to Islam. In Honor of Rémy