Open Access Scientific Reports

Research Article Open Access

Abstract

$ UHODWLYHO\ QHZ DQDO\WLFDO PHPHPHPHPO\WLFD@p € @ W\W WU HV€@ H $W
VLPSOH HI¿FLHQW DQG UHOLDEOH ,Q DGGLWLRQ WKH FDOFXODWLRQV LQYRO
GHPRQVWUDWHG WKDW +'0 LV D SRZHUIXO DQG HI¿FLHQW WRRO IRU)UDFWLRQ
GHPRQVWUDWHG WKDW +'0 LV PRUH HI¿FLHQW WKDQ WKH $'0 $GRPLDQ GHFRPS
PHWKRG +$0 +RPRWRS\ DQDO\VLV PHWKRG DQG +30 +RPRWRS\ GHFRPSRVLW

Keywords: Fractional heat-like and wave-like equations; Homotopy decomposition method; Fractional derivative order

Introduction

Fractional Calculus has been used to model physical and engineering processes, which are found to be best described by fractional di erential equations. It is worth nothing that the standard mathematical models of integer-order derivatives, including nonlinear models, do not work adequately in many cases. In the recent years, fractional calculus has played a very important role in various elds such as mechanics, electricity, chemistry, biology, economics, notably control theory signal, image processing and groundwater problems. In the past several decades, the investigation of travelling-wave solutions for nonlinear equations has played an important role in the study of nonlinear physical phenomena .An excellent literature of this can be found in fractional di erentiation and integration operators were also used for extensions of the di usion and wave equations [1-11].

e solutions of Fractional heat-like and wave-like equations with variable coe cients have attracted attention of many authors in mathematics community. Recently, Shou and He [12] used the variational iteration method (VIM) to solve various kinds of heat-like and wave-like equations. However with VIM one needs rst to obtain, the Lagrange multiplier and the correctional function. In addition of this, sometime, the solutions obtained via the VIM are noisy [13,14] one therefore needs to cancel the noisy term to obtain the correct solution.

Xu and Cang [15] solved the fractional heat-like and wave-like equations with variable coe cients using Homotopy Analysis Method (HAM). e disadvantage of HAM is that, it is very much depended on choosing auxiliary parameter. Momani [13] applied the Adomian Decomposition method to the time fractional heat-like and wave-like equations with variable coe cients. e main disadvantage of the Adomian method is that the solution procedure for calculation of Adomian polynomials is complex and di culty as pointed out by many researchers [16-20].

In this paper, we extend the application of the Homotopy Decomposition Method (HDM) in order to derive analytical approximate solutions to nonlinear time Fractional heat-like and wave-like equations with variable coe cients

e HDM was recently applied to solve: Fractional modi ed Kawahara equation, fractional model of HIV infection of CD4+T cells, attractor fractional one-dimensional Keller-Segel equations, fractional Jaulent-Miodek and Whitham-Broer-Kaup equations;

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Received October 23, 2012; Published January 28, 2013

Citation: Atangana A, Alabaraoye E (2013) Exact Solutions Fractional Heat-
/LNH DQG :DYH /LNH (TXDWLRQV ZLWK 9DULDEOH &R 1:633. doi:10.4172/
VFLHQWL¿FUHSRUWV

Basic Idea of the HDM

To illustrate the basic idea of this method we consider a general nonlinear non-homogeneous fractional partial di erential equation with initial conditions of the following form

( )

$$\left[f(x,t) + L\left(\sum_{n=0}^{\infty} p^n U_n(x,t) \right) + N\left(\sum_{n=0}^{\infty} p^n U_n(x,t) \right) \right] d\tau \quad (3.7)$$

Comparing the terms of same powers of p gives solutions of various orders with the rst term:

( )

Citation: Atangana A, Alabaraoye E (2013) ([DFW 6ROXWLRQV)UDFWLRQDO +HDW /LNH DQG :DYH /LNH (TX
doi:

Citation: Atangana A, Alabaraoye E (2013) ([DFW 6ROXWLRQV)UDFWLRQDO +HDW /LNH DQG :DYH /LNH (TX
doi: VFLHQWL¿FUHSRUWV

RI 'LIIHUHQWLDWLRQ DQG ,QWHJUDWLRQ WR $UELWUDU\ 2UGHU 'RYHU %RRNV 8VH RI 9DULDWLRQ
0DWKHPDWLFV $FDGHPLF 3UHVV 1HZ <RUN 1< 86$ 7UDQVIRUP 0HWKRG DQG $GRPLDQ 'HFRPSRVLWLRQ
2. 3RGOXEQ\ ,)UDFWLRQDO 'LIIHUHQWLDO (TXDWLRQV 0DWKHPDWLFV $FDGHPLF 3UHVV 1HZ
<RUN 1< 86$
3. .LOEDV $$ 6ULYDVWDYD ++ 7UXMLOOR -- 7KHRU\ DQG $SSOLFDWLRQV RI
)UDFWLRQDO 'LIIHUHQWLDO (TXDWLRQV (OVHYLHU $PVWHUGDP 7KH 1HWKHUODQGV
4. &DSXWR 0 /LQHDU 0RGHOV RI 'LVVLSDWLRQ ZKRVH 4 LV DOPRVW)UHTXHQF\
,QGHSHQGHQW ,, *HRSK\VLFDO -RXUQDO ,QWHUQDWLRQDO
0LOOHU .6 5RVV % $Q ,QWURGXFWLRQ WR WKH)UDFWLRQDO &DOFXOXV DQG
)UDFWLRQDO 'LIIHUHQWLDO (TXDWLRQV -KQ :LOH\ 6RQ 1HZ <RUN 86$
6DPNR 6* .LOEDV $$ 0DULFKHY 2,)UDFWLRQDO ,QWHJUDOV DQG 'HULYDWLYHV
7KHRU\ DQG $SSOLFDWLRQV 7D\ORU)UDQFLV *URXS <YHUGRQ 6ZLW]HUODQG
7. =DVODYVN\ *0 +DPLOWRQLDQ &KDRV DQG)UDFWLRQDO '\QDPLFV 2[IRUG
8QLYHUVLW\ 3UHVV 2[IRUG 86$
8. $WDQJDQD $ 1XPHULFDO VROXWLRQ RI VSDFH WLPH IUDFWLRQDO GHULYDWLYH RI
JURXQGZDWHU ÀRZ HTXDWLRQ ,QWHUQDWLRQDO FRQIHUHQFH RI DOJHEUD DQG DSSOLHG
analysis, June 20-24, Istanbul, 20.
6FKQHLGHU :5 :\VV :)UDFWLRQDO GLIIXVLRQ DQG ZDYH HTXDWLRQV - 0DWK
3K\V
10. $WDQJDQD $ 1HZ &ODVV RI %RXQGDU\ 9DOXH 3UREOHPV ,QI 6FL /HWW
11. 2GLEDW =0 0RPDQL 6 $SSOLFDWLRQ RI YDULDWLRQDO LWHUDWLRQ PHWKRG WR
QRQOLQHDU HTXDWLRQV RI IUDFWLRQDO RUGHU ,QW - 1RQOLQHDU 6FL 1XPHU 6LPXO
12. 6KRX '+ +H -+ %H\RQG $GRPLDQ PHWKRG 7KH YDULDWLRQDO LWHUDWLRQ
PHWKRG IRU VROYLQJ KHDW OLNH DQG ZDYH OLNH HTXDWLRQV ZLWK YDULDEOH FRHI¿FLHQWV
3K\VLFV /HWWHUV $
13. 0RPDQL 6KDKHU $QDO\WLFDO DSSUR[LPDWH VROXWLRQ IRU IUDFWLRQDO KHDW OLNH
DQG ZDYH OLNH HTXDWLRQV ZLWK YDULDEOH FRHI¿FLHQWV XVLQJ WKH GHFRPSRVLWLRQ
PHWKRG $SSO 0DWK &RPSXW