Echi, 2:1
http://dx.doi.org/10.4172/scientificreports.607

Open Access Scientific Reports

Reasearch Article Open Access

Abstract

This paper presents an efficient approach for determining the solution of second-order linear differential equation. The second-order linear ordinary differential equation is first converted to a Volterra integral equation. By solving the resulting Volterra equation by means of Taylor's expansion, different approaches based on differentiation and integration methods are employed to reduce the resulting integral equation to a system of linear equation for the unknown and its derivatives the approximate solution of second-order linear differential equation is obtained. Test example demonstrates the effectiveness of the method and gives the efficiency and high accuracy of the proposed

Keywords: Second-order linear differential equation; Volterra integral equation; Taylor's expansion; Error analysis

Introduction

Some second-order differential equations with variable coefficients can be solved analytically by various methods [1]. For general cases, one must appeal to numerical techniques or approximate approaches for getting its solutions [2,3]. The Adomian decomposition method for solving differential and integral equations, linear or nonlinear, has

and integration methods are employed to reduce the resulting integral equation to a system of linear equation for the unknown and its derivatives the approximate solution of second-order linear differential equation is obtained. By studying the estimation of the error give the efficiency and high accuracy of the proposed method [4-6].

Volterra Integrals Equations

We consider the following second-order differential equation :

(E) : y''(t)+p(t) y'(t) y(t) = g(t) (1)

with p,q and g are infinitely differential functions in open interval I ⊂ R. We fix a point a of the interval I. We have, ∀ x ∈ I,

$$\int_a^x \frac{(t-x)^{i+1}}{(i+1)!} y''(t)\,dt = y'(a)\frac{(a-x)^i}{i!} - y'(a)\frac{(a-x)^{i+1}}{(i+1)!} + \int_a^x \frac{(t-x)^{i-1}}{(i-1)!} y(t)\,dt \quad (2)$$

and

$$\int_a^x \frac{(t-x)^{i+1}}{(i+1)!} p(t)\, y'(t)\,dt = -p(a)\, y(a)\frac{(a-x)^{i+1}}{(i+1)!}$$

$$-\int_a^x \Big[\frac{(t-x)^i}{i!} p(t) + \frac{(t-x)^{i+1}}{(i+1)!} p'(t)\Big] y(t)\,dt \quad (3)$$

The differential equation (E) equivalent at integral equation :

$$(E_i): \forall\, x \in I,\ \int_a^x h_{i,x}(t)\, y(t)\,dt = f_i(x)$$

$$y(t) = \sum_{k}^{n-} y^{(k)}(x)\frac{(t-x)^k}{k!} + R_{n,x}(t) \quad (7)$$

where $R_{n,x}(t)$ denotes integral remainder

$$R_{n,x}(t) = \int_x^t \frac{(t-s)^{n-1}}{(n-1)!} y^{(n)}(s)\,ds \quad (8)$$

In particular, if the desired solution y(t) is a polynomial of degree equal to or less than n-1, then $R_{n,x}(t)$=0.

We put for all i and j positive integer 1:

$$b_{ij}(x) = \int_a^x h_{i,x}(t)\frac{(t-x)^{j-1}}{(j-1)!}\,dt \quad (9)$$

For an integer i 1, the function y

us, for all k = 0,…,+ j-2, we have:

$$b_{ij}^{(k)}(a) = 0 \quad (21)$$

So we have:

$$b_{ij}^{(i+j-1)}(a) = (-1)^{i+j} C_{i+j-2}^{j-1} \quad (22)$$

$$b_{ij}^{(i+j)}(a) = (-1)^{i+j-1} C_{i+j-1}^{j-1} p(a) \quad (23)$$

e explore Taylor's approximations, ina of $b_{ij}^{(i+j+1)}$ at ordern-2, give that of b_{ijb}

$$\frac{d^k}{dx^k}\int_a^x \frac{(t-x)^{i+1}}{(i+1)!}g(t)\,dt = (-1)^k \int_a^x \frac{(t-x)^{i+1-k}}{(i+1-k)!}g(t)\,dt \qquad (34)$$

and

$f_i^{(i+2)}$

$$Y_{(n)}(x) = \begin{pmatrix} y_{(n)0}(x) \\ y_{(n)1}(x) \\ . \\ . \\ y_{(n)n-1}(x) \\ \end{pmatrix} \tag{45}$$

From (38) and (41) we have:

$$B_{(n)}(x)\, D_n(x)\, Y_{(n)}(x) \quad F_{(n)}(x)$$

Citation: